# Elements of the Mathematical Theory of Human Systems, Part 3: Mathematics of Cooperation and Confrontation of Human Systems

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**Abstract**

The structural representation of people's life and activity in the form of the equations of state has many applications, one of the most important of which is the quantitative description of the interaction and relationships between human systems.

To construct mathematical models of interactions and relationships between human systems, it is necessary to separate only the sub-flows of mutual actions from the common flow of actions of the parties and present these sub-flows by means of the equations of state.

Sub-flows of actions that represent interactions between human systems can easily be divided into two sub-sub-flows or sequences of actions that individually represent cooperation and confrontation between people.

In turn, the sub-flows of actions representing cooperation and confrontation between human systems can be presented in the form of a system of state equations, on the basis of which mathematical models of interaction between human systems are constructed.

These mathematical models can be used to manage cooperation and confrontation between human systems through purposeful changes in the values of the parameters of the parties.

These problems, which relate to mathematical modeling and control of the behavior of human systems, are discussed in the third part of the paper.

**Key words: Human systems, mathematical theory, cooperation, confrontation, human interaction, state equations, equilibrium, non-equilibrium, human errors.**

## Introduction:

Relations between people and their interactions are the core of the functioning of any human system, and their quantitative representation can have many applications in various areas of organizational science.

Even superficial observations of the behavior and activities of human systems show that relations between people can be in very different states of consent or disagreement.

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On the other hand, the same observations show that human relations are also the result of their mutual actions, which can create a balance or imbalance in the interaction between people.

This means that the universal mathematical model of human actions [1] can be used to quantify interactions between people as sequences of mutual actions.

The first step in this direction is the creation of mathematical models of balanced relations between people, for the implementation of which it is necessary to find quantitative equivalents for such a balance.

Since the balance in human relations also means consent between them, this circumstance indicates the nature of the quantitative equivalent of this consent.

On the other hand, since people having reached an agreement with each other, thereby guaranteeing each other the receipt of some equivalent benefit, the concretization and refinement of this statement can serve as a basis for describing such an equilibrium between people using mathematical equations.

With a more comprehensive examination of the problem, apart from mutual benefit, mutual losses can also be taken into account, since in addition to mutually beneficial stable relations between people there can be stable poor relations, for example between enemies or competitors.

For example, this is a very common case when different countries pursue a common policy directed against each other, but at the same time support a mutually beneficial economic policy.

This can be extended to different areas of the life and activities of human systems, where they simultaneously pursue a policy of cooperation and a policy of confrontation.

This circumstance makes it possible to use the method of state equations for the quantitative description of relations between people and various human systems.

In order to do that, the equations of state can quantitatively represent the benefits and losses of interconnected human systems that have functional links with the results obtained or created by the parties [1], and may also have the form of mutual positive or negative pressures [2].

## Equilibrium and non-equilibrium of life and activity of human systems in light of their state equations

Let us consider the problem of quantitative description of the equilibrium and non-equilibrium states of human systems or associations of people of an arbitrary scale with the help of a parametric representation of their life activities.

It is shown in [1] that at a high level of representation of life and activity of human systems can be described with sufficient completeness by means of four system parameters, namely, the scale or volume of activity $W$, the degree of difficulty of people's actions $D$, the effort $E$ spent in carrying out these activities and the efficiency or average performance of people $P$.

Some of these parameters, namely the scale $W$ and difficulty $D$, reflect the requirements of life for the activities of the human system, and the product of these parameters $W*D$ reflects the degree of complexity of these requirements.

The other part of these parameters, namely the effort E and the efficiency $P$ reflect the ability of people to overcome the difficulties of life, and the product $E*P$ represents a measure of the complexity that the human system can overcome.

According to this approach, the life-support process of human systems can be represented by comparisons and juxtapositions of the complexity $W*D$ of life requirements and the realized complexity of human activities $E*P$.

The point is that comparisons and juxtapositions of the complexities of $W*D$ and $E*P$ are able to adequately represent the equilibrium and non-equilibrium in people's lives.

As for the equilibrium activity of people and human systems, it can be represented in the form of equality of the indicated complexities, i.e.

$$E*P = W*D. \qquad (1)$$

As for the non-equilibrium states, each human system also has a multitude of such states in which either the capabilities and skills of people $E*P$ are less than the requirements of life $W*D$, or these capabilities and skills exceed the requirements of life.

In these cases, taking into account that $E = N*T$ ($N$ - number of people and $T$ - duration of their activity) the behavior of human systems can be represented by inequalities

$$E*P < W*D, \quad \text{or} \quad N*T*P < W*D \qquad (2)$$

and

$$E*P > W*D, \quad \text{or} \quad N*T*P > W*D. \qquad (3)$$

If we consider the above inequalities from the perspective of the development of human systems, then the inequality $N*T*P < W*D$ can be treated as a systematic lag of this system from the requirements of life, and the inequality $N*T*P > W*D$ as a sign of the presence of a certain potential for development.

Moreover, the difference $W*D - N*T*P$, that is, the degree of backlog in the first case and the difference $N*T*P - W*D$, that is, the potential for progress in the second case, can contain valuable information for predicting the behavior of human systems.

So, in the case of one person, the large difference $W*D - N*T*P$ may be an indicator of the uselessness of the personality, and the large difference $N*T*P - W*D$ will speak about outstanding personality qualities.

If we talk about the ethnos, nation, state or civilization, the high value of the difference $W*D - N*T*P$ can be an indicator of their decline and gradual disappearance from the face of the earth, and the large difference $N*T*P - W*D$ can serve as a good indicator of the ability of an ethnos or nation to develop.

If we consider the strong inequality $T*P >> W*D$ in the case of an individual, then it can become a platform for the analysis of rare phenomena such as the genius of people, the foundations of religions and empires, and other rare phenomena such as "black swans" [3].

**People's actions as a source of influence on others**

People's actions are the source of everything, including material wealth, spiritual values, love, hate, security, danger, conflicts between human systems, etc.

From the point of view of relations between people, their actions, as sources of influence on others, can be interpreted as positive and negative pressure on them [2].

Manifestations of positive pressure or simply pressure on others are censure, disapproval, punishment and misunderstanding of the actions and doings of others.

Manifestations of negative pressure are the encouragement, approval and understanding of the actions and doings of others.

It is often possible to observe how the thoughts, approaches and actions of people coincide and are in agreement on some issues, and, conversely, such consent is absent on other issues.

In the first case, we can talk about a partial equilibrium in the relationships between people, and in the second case - about a partial non-equilibrium between them.

In the case of equilibrium in all sectors, one can speak of a global equilibrium or harmony in the relationships between people.

**Equations of state and quantitative description of human interactions**

The interactions of people are the foundation on which the life of all human systems, including the family, the work of companies and firms, countries and states, is based.

Therefore, a quantitative description of the activities of human systems cannot be imagined without mathematical models of human interactions.

At the heart of all mathematical models and theories of human interactions should lie that simple and obvious idea that every action of a person, if it is connected with other people, causes some opposition.

On the other hand, interpersonal relations and daily interactions of people are carried out in the form of a sequence of their individual actions [1].

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This means that on the basis of state equations of the people's actions and activities, it is also possible to build quantitative models of interpersonal relations and, ultimately, the mathematical theory of interactions between people and human systems.

The construction of such mathematical models and the theory of human systems is of exceptional importance, since they can serve as a springboard for the construction of quantitative methods for describing the life activity of larger-scale human systems, including the description of the companies and firms, various organizations, countries and states, world order and international relations, etc.

The fact that people's actions can be regarded as the source of benevolence, encouragement, love, resentment, hatred, punishment or pressure, allows us to use the equations of state to quantify the human relations.

A person for an arbitrary period of time makes a certain number of actions and produces various material and non-material results and corresponding values, including means for maintaining equilibrium with the environment.

In addition, the mechanism of maintaining a balance between people implies a balance between their mutual support or confrontation.

Both support and opposition between people are realized through actions that produce positive and negative generalized pressures on the opposite side.

If from the flow of $n$ actions performed by a person within the period of time $T$ to allocate an $i$-th action, then at the equilibrium between the requirements of life and the possibilities of man this action according to equation (1) can be represented by the following equation of state [1]

$$T_i * P_i = W_i * D_i , \quad (4)$$

where $T_i$ - is the duration of the mentioned action, $P_i$ - is the human's performance, $W_i$ - is the size of the action and $D_i$ - is the difficulty of the action implementation.

Summarizing the balance equation (4) for all $n$ actions in a time interval $T = \sum_1^n (T_i)$, one can obtain the equation of state for human activity for the mentioned time interval

$$\sum_1^n (T_i * P_i) = \sum_1^n (W_i * D_i) . \quad (5)$$

To continue the consideration of the problem, let us analyze the case of the equilibrium coexistence of two human systems. In the simplest case, we select from the equation of state (5) only the members of mutual support and encouragement, as well as the terms reflecting the conflict and confrontation between these systems.

Thus, the incentive or benevolent actions of the first human system in relation to the second will be represented by the following equation of state

$$T_{12enc} * P_{12enc} = W_{12enc} * D_{12enc}, \qquad (6)$$

where $T_{12enc}$ - is the total time spent on incentive or benevolent actions, $P_{12nc}$ - is the effectiveness of these actions, $W_{12enc}$ - is the volume of actions, $D_{12enc}$ - is the degree of difficulty in carrying out these actions, and the index $enc$ means encouragement.

The expression of discontent and confrontation of the first human system in relation to the second can be represented by the equation

$$T_{12pan} * P_{12pan} = W_{12pan} * D_{12pan} \qquad (7)$$

where the index $pan$ means punishment, and the remaining terms of the equation have the same meaning as in equation (6), only in the negative sense.

The quantities $W_{12enc}$ and $W_{12pan}$ in the equations (6) and (7) can be interpreted as sources of positive and negative actions of the first human system with respect to the second.

Assuming that in the simple case there is a linear dependence between the first human system's encouragement $X_{12}$ for the second system and the volume of actions $W_{12enc}$, we will have

$$X_{12} = k_{X12} W_{12enc} , \qquad (8)$$

where the coefficient $k_{X12}$ indicates the strength or intensity of the encouragement.

A similar relationship exists also for the magnitude of the negative attitude of the first human system to the second, that is,

$$Y_{12} = k_{Y12} W_{1pan}, \qquad (9)$$

where the coefficient $k_{Y12}$ indicates the strength or intensity of the mentioned negative attitude.

Hence, taking into account that $W_{12enc} = \frac{X_{12}}{k_{X12}}$, and $W_{12pan} = \frac{Y_{12}}{k_{Y12}}$ and substituting them into equations (6) and (7), we obtain new equations of state for $X_{12}$ and $Y_{12}$

$$X_{12} = \frac{k_{X12} * T_{12enc} * P_{12enc}}{D_{12enc}}, \qquad (10)$$

$$Y_{12} = \frac{k_{Y12} * T_{12pan} * P_{12pan}}{D_{12pan}}. \qquad (11)$$

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Similarly, one can obtain the following equations of state for the positive and negative attitude of the second human system to the first

$$X_{21} = \frac{k_{X21} * T_{21enc} * P_{21enc}}{D_{21enc}}, \qquad (12)$$

$$Y_{21} = \frac{k_{Y21} * T_{21pan} * P_{21pan}}{D_{21pan}}. \qquad (13)$$

Thus, the resulting pressure, that is, the sum of the positive and negative pressures of the first human system in relation to the second will be $Y_{12} - X_{12}$, and the total pressure of the second human system with respect to the first system will be $Y_{21} - X_{21}$.

The equilibrium between these two human system can be expressed by the equation

$$Y_{12} - X_{12} = Y_{21} - X_{21}. \qquad (14)$$

After the corresponding substitutions from the equations of state (10), (11), (12) and (13) into the equilibrium condition (14), we obtain

$$\frac{k_{Y12} * T_{12pan} * P_{12pan}}{D_{12pan}} - \frac{k_{X12} * T_{12enc} * P_{12enc}}{D_{12enc}} = \frac{k_{Y21} * T_{21pan} * P_{21pan}}{D_{21pan}} - \frac{k_{X21} * T_{21enc} * P_{21enc}}{D_{21enc}}. \qquad (15)$$

This is the equation of the state of the relationships between two human systems, which in general contains mutual benevolent actions in the form of encouragement and mutual malicious actions in the form of punishments, which may have extreme manifestations in the form of uprisings and wars, and so on.

If between the specified human systems there are only warm and friendly relations, then the equation of state of such a human system or the equation of cooperation between them will have the following form

$$\frac{k_{X12} * T_{12enc} * P_{12enc}}{D_{12enc}} = \frac{k_{X21} * T_{21enc} * P_{21enc}}{D_{21enc}}. \qquad (16)$$

In the case of confrontation between these systems $X_{12} = 0$ and $X_{21} = 0$, and then the equation of state of their interrelations will have the form

$$\frac{k_{Y12} * T_{12pan} * P_{12pan}}{D_{12pan}} = \frac{k_{Y21} * T_{21pan} * P_{21pan}}{D_{21pan}}. \qquad (17)$$

All the resulting equations that reflect the states of cooperation and confrontation between human systems can be applied to solve numerous problems of analysis, synthesis and optimization of people's relationships from interpersonal relations to the level of international relations.

The randomness factor of some parameters in these equations will allow us to assess the risks of conflicts between human systems, which, of course, is a critically important area of research.

It is characteristic that the equations of cooperation between human systems, and the equations of confrontation between them, have the same structure and mathematical form, which is not surprising, since they are expressions of the coexistence of people.

Let us repeat that the structural identity of the above-mentioned equations of state should not become a source of misunderstanding and confusion, since the same parameter in different equations carries different semantic loads, describing completely different objects and phenomena.

**Expansion of the possibilities of the equations of interaction of human systems taking into account the intensity of actions of the parties and human errors**

The resulting equations of the state of cooperation and confrontation of human systems can be expanded and deepened in different areas and directions of human activity.

In particular, from a practical point of view it is very important to be able to take into account in these equations such parameters of human systems as the intensity of their interactions, the number and professional qualities of interacting people, their errors and much more.

To take into account these factors, we make use of the functional relationship between productivity $P$ and the intensity $I_{Max}$ of human actions [4]

$$P = \frac{I_{Max}}{1 + a * t_a}, \tag{18}$$

where $a$ - is the average error rate of humans, and $t_a$ - is the average time to correct one error.

Taking into account formula (18) and the quality $Q$ of people's actions [1], we get a more powerful and versatile equation for describing the relationships of human systems

$$T * M * I_{Max} = Q * W * D(1 + a t_a). \tag{19}$$

Thus, taking into account the last relation, the general equation of the state of interaction of human systems (15) takes the following form:

$$\frac{k_{Y12} * T_{12pan} * M_{12pan} * I_{12pan}}{Q_{12pan} * D_{12pan}(1 + a_{12pan} t_{12apan})} - \frac{k_{X12} * T_{12enc} * M_{12enc} * I_{12enc}}{Q_{12enc} * D_{12enc} * (1 + a_{12enc} t_{12aenc})} =$$

$$= \frac{k_{Y21} * T_{21pan} * M_{21pan} * I_{21pan}}{Q_{21pan} * D_{21pan}(1 + a_{21pan} t_{21apan})} - \frac{k_{X21} * T_{21enc} * M_{21enc} * I_{21enc}}{Q_{21enc} * D_{21enc} * (1 + a_{21enc} t_{21aenc})}. \tag{20}$$

Accordingly, the equation of cooperation and friendly relations between human systems (16) will have the following form

$$\frac{k_{X12} * T_{12enc} * M_{12enc} * I_{12enc}}{Q_{12enc} * D_{12enc} * (1 + a_{12enc} t_{12aenc})} = \frac{k_{X21} * T_{21enc} * M_{21enc} * I_{21enc}}{Q_{21enc} * D_{21enc} * (1 + a_{21enc} t_{21aenc})}, \quad (21)$$

and the equation of confrontation and unfriendly relations between people (17) takes the form

$$\frac{k_{Y12} * T_{12pan} * M_{12pan} * I_{12pan}}{Q_{12pan} * D_{12pan}(1 + a_{12pan} t_{12apan})} = \frac{k_{Y21} * T_{21pan} * M_{21pan} * I_{21pan}}{Q_{21pan} * D_{21pan}(1 + a_{21pan} t_{21apan})}. \quad (22)$$

The equations obtained have numerous applications in the analysis of the relationships and interactions of human systems. In particular, the last equation (22) can serve as a starting point for the analysis of competition between companies, as well as for the modeling of the arms race between countries.

Moreover, if we take into account that the intensities of interactions and errors of human systems directly depend on the number of people participating in these processes, we can further expand the possibilities of analysis in applying the equations obtained to geopolitical problems.

**Future research**

Further studies of the issues discussed in this paper have several developmental tendencies that are related to the mathematical modeling of equilibrium and non-equilibrium states of relationships between people.

First of all, it refers to the change management in equilibrium human relations, changes, depending on the size of which human relations can have different ramifications in the form of bifurcations.

Here, in one case, we are talking about small changes of some parameters characterizing the relationships of human systems, which leads to nonlinear changes in other system parameters, the forecasts and estimates of which are necessary for the analysis of the stability of systems.

In parallel with the minor changes that are of a massive nature, in the interrelationships of human systems there are rarely large changes that push the parties to the unstable non-linear areas of their activity and behavior, the study of which is important from the point of view of conflict situations in the relations between the parties.

The results obtained in the work refer to the equilibrium interactions between human systems, but within the framework of the considered approach, the subject of investigation can also be non-equilibrium phenomena in the cooperation and confrontation of these systems which is an extremely important area of research.

## Conclusions

1. The most important conclusion that can be drawn from this work is that on the basis of the universal equation of the state of human actions it is possible to construct mathematical models of cooperation and confrontation of human systems.

2. These models can have different applications, from the analysis of ordinary household relations between people to the investigation of international relations by analytical means.

3. Taking advantage of the fact that parameters reflecting high-level human activities have many functional links with parameters reflecting lower levels of the hierarchy of people's life and activity, it is possible to expand the possibilities and spheres of influence of the equations of state in different directions.

4. Expanding the possibilities of the equations of state, reflecting different aspects of people's lives, it is possible to find out new functional relationships between various aspects of the activity of human systems in an analytical way.

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**Dr. Pavel Barseghyan** is a consultant in the field of quantitative project management, project data mining and organizational science. Has over 45 years' experience in academia, the electronics industry, the EDA industry and Project Management Research and tools development. During the period of 1999-2010 he was the Vice President of Research for Numetrics Management Systems. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of quantitative project management, mathematical theory of human work, electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia). Pavel's publications can be found here: http://www.scribd.com/pbarseghyan and here: http://pavelbarseghyan.wordpress.com/. Pavel can be contacted at terbpl@gmail.com