Elements of the Mathematical Theory of Human Systems

Part 2: Structural Mathematical Models of the Life of Humans Based on the Method of State Equations[1]

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Abstract

Quantitative representation of the structure of the life and activity of humans is one of the central problems of the mathematical theory of **Human systems**.

The fact that human activity consists of separate actions allows us to build a mathematical theory of **Human systems** on the basis of quantitative models of people's actions.

The first step in constructing such a theory is a structural representation of human activity in the form of mixed networks of the sequences of actions that make up the content of the people's life process.

This approach allows us by using the equation of state of human actions to develop a quantitative description of **Human systems** of arbitrary complexity in the form of a system of state equations.

The second part of the paper is devoted to the structural representation of human life and activity as a set of flows of people's biological and social actions.

At the heart of the mathematical model of human life and activity lies the fundamental statement that the biological and social aspects of human life are conditioned by each other and constitute a single whole.

Introduction: Problems of practical applications of the mathematical theory of human systems

Quantitative description of the behavior and activities of people and human systems from single conceptual positions is crucial for organizational science, which, in turn, can have a variety of economic, political, geopolitical and other applications.

The point is that the present state of organizational science, with its fragmented and segmental methods, needs the development of such scientific theories and mathematical models that, by

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virtue of their generality, can consider the quantitative description of the lives of people in different political, economic, social, business, religious , moral and other conditions, as particular applications of the same scientific approach [1].

To develop such a scientific approach, first of all, it is necessary to search for and find in the various organizational forms of people's lives such common features or such a commonality on the basis of which it would be possible to build some universal theory of human systems.

From the point of view of quantitative description, the obvious commonality of the various organizational forms of people's association under study is that they are all systems for which the main provisions of the general theory of systems are applicable.

In addition, these organizational forms of uniting people have another specific commonality, namely, that they are all human systems with characteristic internal laws of a biological and social nature, at the center of which are the psychology and behavior of people with their various manifestations.

In short, all organizational forms of uniting people, as well as different societies, be they totalitarian, authoritarian or based on democratic principles, basically are based on the same human psychology, which simply has different manifestations under different conditions.

Continuing the search for commonalities in the behavior and activities of people, we can see that they all consist of individual actions of people, each of which have a magnitude and size, the complexity of implementation, and for their implementation are required effort, time, knowledge and skills of people [2].

This means that in order to develop a universal mathematical theory of human behavior and activity, it is first necessary to develop a quantitative theory of individual actions of people of a universal nature as a first step in this direction.

The second step towards developing a mathematical theory of human activity is the development of quantitative methods of transition from describing people's actions to describing their activities.

To develop a mathematical theory of people's activities on the basis of a quantitative description of their actions, the structural representation of people's activities in the form of various combinations of flows and successive chains of their actions is of great importance.

Structured representation of the activity of human systems in the form of a set of different action flows

Since human life is a sequence of actions, the life path of people at an arbitrary time interval T can be represented in the form of a simple sequence of actions, presented in Fig.1.

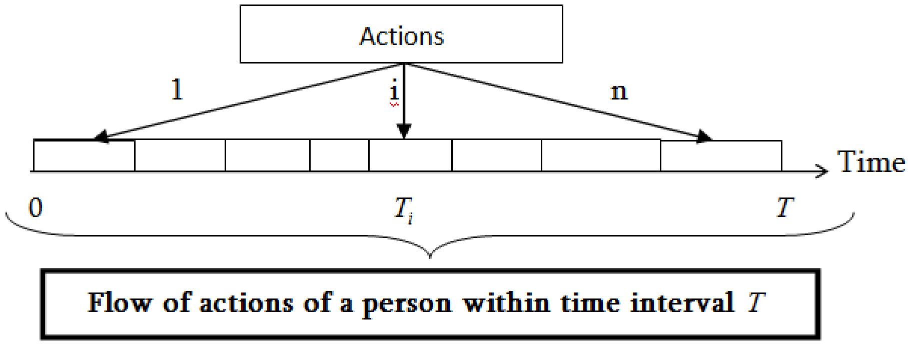

Fig.1 Time sequence of human actions

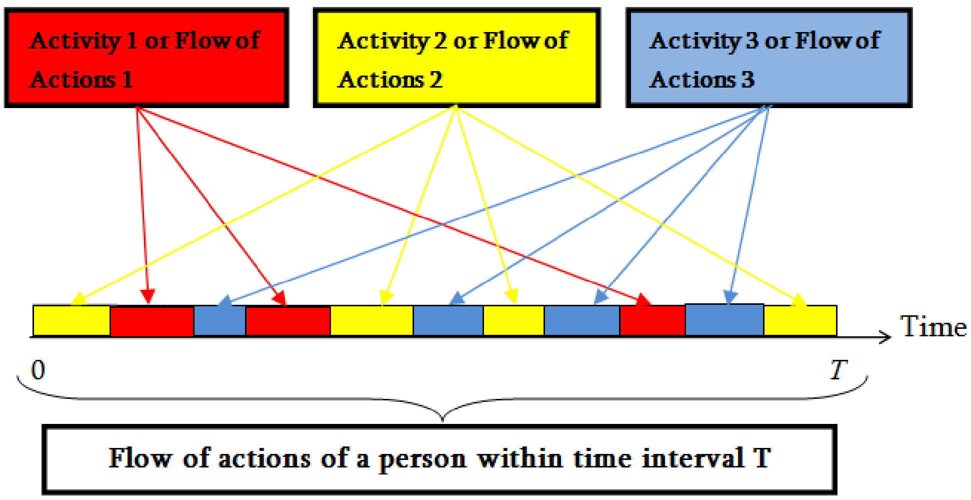

Fig.2 Splitting the general flow of human actions into sub-flows for three different activities

In cases where a person is engaged in more than one type of activity, then from the general flow of his/her actions, sub-flows can be identified and considered as separate activities (Fig. 2).

Regardless of the fact that actions in Fig. 2 are of a sequential nature, these sub-flows of actions are parallel activities carried out by a person (Fig. 3).

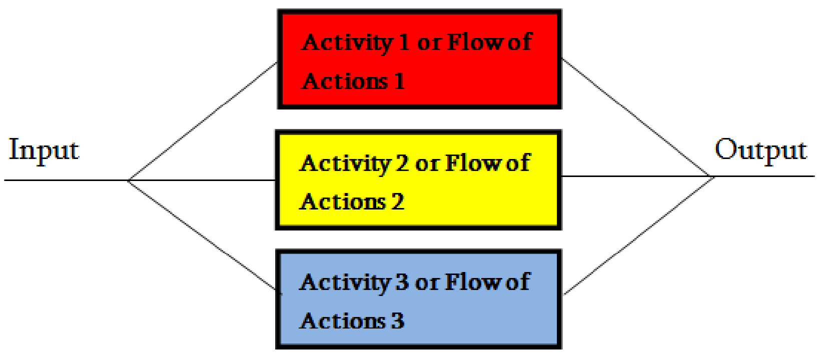

Fig.3 Separate sub flows of actions can be presented as parallel activities of a person

Guided by this logic, human life and activity can be represented in the form of a network of different sub flows of human actions that can be used to build mathematical models of people's lives and the behavior of human systems.

The basis of all this is that every human action, like any of its sub-activities, can have its own equation of state, and the life and activity of people quantitatively can be represented as a system of state equations.

The same philosophy and logic also act in the quantitative description of the behavior and activity of human systems of any complexity. including groups of people and organizations, states and countries, world order, geopolitics and international relations, etc.

Representation of human life as the sum of two flows of biological and social actions

To illustrate the possibilities of the theory for analyzing the behavior of human systems, let us consider the division of the life flow of people's actions into biological and social streams of actions [3].

If from the flow of n actions performed by a person in a period of time T to allocate an i-th action, then at the equilibrium between the requirements of life and the possibilities of man this action can be represented by the following equation of state [2]

$$T_i * P_i = W_i * D_i, \qquad (1)$$

where T_i - is the duration of the mentioned action, P_i - is the human's performance, W_i - is the size of the action and D_i - is the difficulty of the action implementation.

Summarizing the balance equation (1) for all n actions for the time interval $T = \sum_1^n (T_i)$, one can obtain the following equation of state that describes the human activity

$$\sum_1^n (T_i * P_i) = \sum_1^n (W_i * D_i). \qquad (2)$$

Assuming that the m actions of the total number of n actions were of a biological nature, and the rest $n-m$ - a social character, then the equation of state (2) will be split into two equations of state, one of which will reflect the balance of the biological side of human life, and the second - the balance of the social side of human life:

$$\left.\begin{aligned} &\sum_1^m (T_{bi} * P_{bi}) = \sum_1^n (W_{bi} * D_{bi}), \qquad (3\text{-}1) \\ &\sum_1^{n-m} (T_{si} * P_{si}) = \sum_1^n (W_{si} * D_{si}). \qquad (3\text{-}2) \end{aligned}\right\} \qquad (3)$$

Here the indices b and s correspondingly denote biological and social.

For transparency and clarity of analysis, let us consider those idealized and simplest cases of the problem when dealing with homogeneous and similar to each other actions of a biological nature and with the similar actions of a social nature.

This makes it possible to bring the equations of the biological and social states of man (3) to the next glacially simplified form

$$\left.\begin{aligned} P_b\sum_1^m T_{bi} &= D_b\sum_1^n (W_{bi}), \qquad (4\text{-}1)\\ P_s\sum_1^{n-m} T_{si} &= D_s\sum_1^n (W_{si}), \qquad (4\text{-}2)\end{aligned}\right\} \qquad (4)$$

where P_b, D_b, P_s and D_s are the averaged parameters of the two sub flows of actions.

Taking into account that the magnitude $\sum_1^m T_{bi}$ is the total duration T_b of human biological actions, and the magnitude $\sum_1^{n-m} T_{si}$ is the total duration T_s of social actions and that the sums $\sum_1^n (W_{bi})$ and $\sum_1^n (W_{si})$ accordingly are the total magnitudes of biological activities W_b and the amount of social activity W_s of a person, instead of the system of equations (4) we get

$$\left.\begin{aligned} P_bT_b &= D_bW_b, \qquad (5\text{-}1)\\ P_sT_s &= D_sW_s. \qquad (5\text{-}2)\end{aligned}\right\} \qquad (5)$$

Here the equation (5-1) represents the balance between the biological requirements of life D_bW_b and the ability P_bT_b of a person to perform the corresponding biological functions, and the equation (5-2) is the balance between the social requirements of life D_sW_s and the ability of a person P_sT_s to perform the corresponding social functions.

The external similarity of the equations obtained should not create confusion, since they differ in content and reflect completely different life support functions of humans.

The extensions and ramifications of these equations (5-1) and (5-2) in different directions serve as the basis for all further analysis.

Examples of constructing mathematical models of human life based on the equations of state

Depending on the specific goals of modeling the life activity of human systems, the separation of the total flow of its actions into sub-flows of different type of actions can have different directions and degrees of detail.

For example the system of equations (5) can be expanded and deepened in different directions, which for equation (5-1) can reflect the safety and energy functions of living systems, and for the equation (5-2) - purely social functions.

If, for example, a person studies and works simultaneously, the equation of state of the social side of his/her life will be split into two equations that describe the balanced state of learning and work processes of the person.

In this case, human life is described by the following three equations of state:

$$\left.\begin{aligned} &P_b T_b = D_b W_b, && (6\text{-}1)\\ &P_{sL} T_{sL} = D_{sL} W_{sL}, && (6\text{-}2)\\ &P_{sW} T_{sW} = D_{sW} W_{sW}, && (6\text{-}3) \end{aligned}\right\} \quad (6)$$

Here the indices L and W correspondingly denote Learning and Work.

To further detail the mathematical model of a person's life, one can divide the biological sub-flow of human actions into sub-sub-flows of actions to ensure safety and obtain energy or food.

On the other hand, if a person, apart from teaching and work, also engages in political activities, his/her social life will be presented as a sum of three types of action streams.

This means that in this particular case the human condition can be described with the help of five equations: two equations for reflecting his/her state of security (Seq) and energy (E), and three equations for learning (L), work (W) and political activity (Pol).

$$\left.\begin{aligned} &P_{bE} T_{bE} = D_{bE} W_{bE} && (7\text{-}1)\\ &P_{bSeq} T_{bSeq} = D_{bSeq} W_{bSeq} && (7\text{-}2)\\ &P_{sL} T_{sL} = D_{sL} W_{sL} && (7\text{-}3)\\ &P_{sW} T_{sW} = D_{sW} W_{sW} && (7\text{-}4)\\ &P_{sPol} T_{sPol} = D_{sPol} W_{sPol} && (7\text{-}5) \end{aligned}\right\} \quad (7)$$

In addition to this, for a more detailed and deepened description of the human life, it is possible to present with the aid of separate equations the equilibrium conditions of human's financial state, state of health, mental state, etc.

The peculiarity of all these equations is that the parameters of a person's state are related to each other with functional relations, due to which the system of equations (7) is one whole, and for this reason it becomes possible to construct an integral quantitative picture of a person's condition and life.

Derivation of the functional relationship between the income of a person and the level of his/her satiety from the equations of state

Consider the application of the system of state equations (5) to establish the functional relationship between the level of satiety and the person's income.

For this purpose, equation (5-1) can be interpreted as a mathematical model of food intake and renewal of the energy resources of a hungry person to a satiated state, and equation (5-2) - as a model of a person's arbitrary activity, as a result of which he earns money for food , but at the same time consumes energy and becomes hungry.

In other words, equation (5-2) describes the expenditure of energy and making money by a person, and the equation (5-1) is the opposite - spending money on food and restoring the energy resources of the body.

With a more detailed interpretation of the equation of eating (5-1), its left side, that is, the value T_b , is the duration of food intake, and the value P_b is interpreted as the intensity of eating, that is, the amount of calories consumed per unit of time. This means that the left side of this equation is the amount of food, the necessary amount of money Z for the acquisition of which is determined by a simple formula

$$Z = k_z P_b T_b \,, \quad \text{or} \quad \frac{Z}{k_z} = P_b T_b \,, \qquad (8)$$

where k_z - is the cost of the unit of food.

In the right-hand side of the equation (5-1), we have the magnitude W_b of the actions for obtaining and accepting food and the difficulty D_b accompanying the process of obtaining and accepting food. Since the result of all these actions is satiety, we can assume that the result of the activity W_b is satiety F , or, in the terminology of the mathematical theory of human systems, the adoption of food produces satiety.

Assuming a linear connection between the quantities W_b and F we have

$$F = k_F W_b \,, \ \text{or} \ W_b = \frac{F}{k_F} \,, \qquad (9)$$

where the coefficient k_F reflects the calorie content of the food.

It is assumed that the only purpose of eating is to restore the energy consumed, while eliminating nonlinear phenomena such as gluttony.

Substituting the values of $P_b T_b$ and W_b into the equation (5-1), we obtain a new state equation for food intake and restoration of the person's energy resource, which contains the cost of food Z and the degree of satiety F.

$$\frac{Z}{k_z} = D_b \frac{F}{k_F} \text{ or } k_F Z = k_z D_b F \text{ or } Z = \frac{k_z D_b}{k_F} F \text{ or } F = \frac{k_F}{k_z D_b} Z \qquad (10)$$

The simple meaning of the resulting equation of state (10) is that satiety costs money and a decrease in income can lead to starvation.

The unfolded meaning of the equation of state (10) is that the degree of satiety F is directly proportional to the calorie content of the food k_F and the amount of money Z intended for the purchase of food and inversely proportional to the value of the unit of food k_z and the difficulties D_b accompanying the process of obtaining and taking food.

Let us now turn to the interpretation and analysis of the social state equation (5-2), which describes the expenditure of energy accompanying human's activity and making money, the portion of which is intended to purchase food and restore human's energy reserves.

This equation describes any occupation of a person who, acting with efficiency P_s over time T_s and overcoming difficulties D_s, performs work W_s for which he receives a certain amount of money.

Assuming a linear relationship between the magnitude of the work W_s and the reward for it Z_0, we will have

$$Z_0 = k_{F0} W_s \text{, or } W_s = \frac{Z_0}{k_{F0}}, \qquad (11)$$

where k_{Z0}- is the earning for the unit of work.

Substituting the value W_s from (11) into the equation of state (5-2), we obtain

$$P_s T_s = D_s \frac{Z_0}{k_{Z0}} \text{, or } k_{Z0} P_s T_s = D_s Z_0 \text{, or } Z_0 = \frac{k_{Z0} P_s T_s}{D_s}. \qquad (12)$$

Part of the salary in quantity

$$Z = k_{\%} Z_0 = \frac{k_{\%} k_{Z0} P_s T_s}{D_s} \qquad (13)$$

is intended to purchase food and therefore becomes a bridge between equations (5-1) and (5-2), that is, a bridge between earning money and human satiety.

The resulting expression (13) in itself is a certain equation of state connecting the budget for food with the conditions of life and work of a person.

Hence, substituting the value Z from equation (13) into the equation (10), we obtain the equilibrium condition between the salary Z_0 and the level of satiety F of a person.

$$F = \frac{k_{\%} k_F}{k_z D_b} Z_0 . \qquad (14)$$

Substituting the value of Z_0 from (12) into the equation (14), we obtain a functional relationship between the level of satiety and various conditions of life and human activity.

$$F = \frac{k_{\%} k_F}{k_z D_b} \frac{k_{Z0} P_s T_s}{D_s} . \qquad (15)$$

Taking into account that the ratio $\frac{P_s T_s}{D_s}$ is the volume of activity W_s we get

$$F = \frac{k_{\%} k_F k_{Z0}}{k_z D_b} W_s . \qquad (16)$$

The obtained results (14), (15), and (16) are also equations of state of humans containing such system parameters as satiety (hunger) level F, salary Z_0, difficulty D_b in obtaining and making food, payment k_{Z0} per unit of activity or work, caloric value k_F of food, the cost k_z of a unit of food, the share $k_{\%}$ of wages for food, the efficiency of the activity P_s, the duration of the activity T_s, the difficulty of the activity D_s and the volume W_s of activity.

Thus, by splitting the basic equation of the state of a person or the human system into the new equations of state, new possibilities for estimating and predicting quantitative parameters characterizing the life and behavior of people are opened up.

The result obtained above is one of the particular reflections of the fact that the biological and social aspects of people's lives are mutually conditioned.

Problems, the solution of which can be reduced to the considered model of consumption and renewal of energy by a person

The considered mathematical model of the cycle "work - fasting and fatigue - eating food - the renewal of strength and energy by a person" is a typical example of alternating and mutually conditioned phenomena in the life and activities of people or the human system.

According to the proposed model, if the activity of the human system is based on the use of different resources, the process of spending which generates revenue, part of which is directed toward the acquisition of new batches of resources, then mathematically this reduces to a system of algebraic equations of state.

Unlike the well-known methods of supply chain modeling and resource management [4], the present work takes into account the characteristics of people as parts of the system as well.

This is true for any type of resources, including financial, human, natural and even ideological and spiritual resources.

Conclusions

1. Any kind of human activity can be represented as a sequence of actions, which also means that mathematically it can be presented by some equation of state;
2. If a person participates simultaneously in different types of activities, then this behavior can be structured in the form of different sequences of actions pertaining to one or another type of activity;
3. This will mean that the life and activities of a person acting in different areas can be mathematically represented in the form of a system of state equations;
4. The external similarity of these equations should not cause confusion, since the internal contents of these equations are radically different from each other;
5. The fact that the parameters and variables in these equations are in mutually functional relations and, accordingly, are conditioned by each other, allows us to construct a consistent quantitative picture of human life and activity;
6. Moving towards further fragmentation of human life and activity in the form of a detailed system of action sequences of humans, we are able to achieve a more meaningful quantitative description of people's lives;
7. In particular, human life and activity can be viewed as a combination of two streams of biological and social actions;
8. With this approach, life and human activity can be represented in the form of a system of two equations, the solutions of which will allow to evaluate the results of human activities and, as a consequence, the quality of life;
9. The discussion on a relatively simple problem in this article is aimed at showing that with the help of the proposed approach it is possible to solve problems of quantitative description of the behavior of larger-scale human systems, since such a description always reduces to the representation of their structure in the form of a system of state equations.
10. Based on the presented mathematical model of human behavior and activity, it is possible to create various software tools that can be used for analysis, forecasting and evaluation, as well as for making decisions and management of various human systems.

Continuation of work

Mathematical theory of human systems also provides an opportunity to describe the relationships of people with the help of mathematical models and equations, whether it be the everyday relations between people, business relations at the level of organizations or international relations.

For the construction of mathematical models of the relationships of human systems, it is necessary to separate from the general flow of actions of the interacting parties only those sub-streams of actions that relate to each other and each of sub-streams will have its own equation of state.

In addition, further fragmentation of the sub-streams of relationships can divide them into sub-sub-streams of actions of the interacting parties representing by them the cooperation and confrontation of human systems.

Each of cooperation and confrontation sub-sub-flows can be described in terms of the equation of state, on the basis of which mathematical models of the relationships between human systems are constructed and which will be discussed in the third part of the article.

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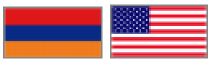

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