Modeling the project management tasks under the risk and uncertainty conditions with a cyclic alternative network model

By L.I. Averbach, V.I. Voropayev, J.D. Gelrud

The high degree of complexity of making the implementation plans of a large number of works by many participants in the project, taking into account a wide range of resources used, the need for systematic monitoring of their implementation and adjustments often require effective methods for solving this complex class of problems.

This paper addresses the problem of project planning and control, as a set of interrelated activities, taking into account the risk and uncertainty of the conditions for their performance.

Endowed with similar characteristics are different types of projects in different areas of activity:

- expansion, modernization, and diversification of production
- construction of buildings and structures
- design and manufacture of a new product sample
- preventive maintenance of complex equipment
- installation and commissioning of computer network, or other complex and diversified equipment
- research and design
- construction and repair of ships
- manufacture and assembly of a large single product, etc

The main reason that such projects are often late in completion and lack control is the fact that these projects include many different random effects. This reflects in random duration of work, the structure of the model with random parameters unknown in advance, etc.

However, project managers tend to avoid "probabilistic" terminology and try to control complex, highly uncertain projects based only on deterministic models and methods, which causes systematic errors in estimating the parameters of the project, particularly, the time of its execution, which in most cases is cut short. This allows us to conclude that the development of sound models and stochastic project management is a relevant and important theoretical and applied task. [1]

Besides, the mathematical models are very diverse both in terms of their adequacy to the described processes and the complexity of perception and results. This is an important point because as it is rightly pointed out in [2]: "The manager would rather not resolve the issue than have models of which he does not have a clue."

The development of modern project management methods began in the late 1950s with the advent of the first articles on network modeling [3], which introduced the so-called
traditional (or classic) network models. At the heart of the first models in project management that used network (PERT, CPM) was the "critical path method" - a powerful but basically very simple method of analyzing the planning and scheduling work distribution for complex projects. This method makes it possible to determine a/ which tasks are "critical" in their impact on the overall duration of the project calendar and b/ how to make up the best schedule of all activities in the project in order to achieve the target date at minimal cost.

By the end of 1960s V.I. Voropayev and his team had worked out the generalized network models (GNM) [4], which present an opportunity to apply a wide range of technological dependencies.

These models make it possible reflecting such relationships between the activities of a project as the combination of performance and sustainability, considering the variable intensity of their implementation, and reflecting the model dependence and restrictions of types "no earlier than" and "no later" for the project as a whole, individual activities and, most importantly, for the parts of the activities. Generalized network models make modeling technological processes more accurate while managing a complex project than the traditional network, and they can significantly decrease detailing the description of the simulated object without losing the value and reliability of information. This makes it possible to "serve" the various hierarchically structured categories: the organizational structure of the project management, the process structure, the structure of resources, the unit of time (year, month, week, day, and hour), etc. Such models are of particular importance in making decisions on optimization plans with various criteria relating to the use of resources and compliance with specific technological and organizational requirements. It is imperative that the time interval between the given pairs of activities not exceed the predetermined value. For example, such requirements include the continuity of carrying out the work on the project, the continuity or limit between tasks, the absolute limiting of the terms of some jobs, etc.

At the same time the aforementioned team developed the probability network models and corresponding statistical techniques to solve scheduling problems based on traditional network models with the probability nature of the parameters of the managed object. [5] It is worth mentioning that the first traditional network models (RERT) took into account the probability nature of the work duration, but at the same time they built the determined models which contained mathematical expectations of the work duration, assuming they meet the $\beta$-distribution. It is not always justified to conduct a seemingly natural analysis of the probability problems, i.e. replacing the random parameters by their mean values and calculation of optimal plans based on the determined problems so obtained. When the parameters of the problem are averaged, there occurs a violation of the adequacy of the model to the managed object. The resulting solution of the problem by determined average parameters does not often meet the objectives for different parameters of restrictions. Therefore, when using the probability network models, it is necessary to find an acceptable solution exceeding an afore-specified number of $\alpha > 0$ (in general, $\alpha$ is a vector of confidence intervals for the implementation of those cases in which possible discrepancies in certain limitations cause different damage).

The next stage in developing the model and methodological support of project management systems is stochastic network models (GERT, VERT, CAAN, GAAN), in
which activities and events are not fixed. They maintain a logical relationship of a very complex structure, present a means to reflect both a wide range of relationships between the activities and events and the branching directions of the project realization with many alternatives. They also allow for the presence of random effects, circumstances, and obstacles [6]. A stochastic network model shows a process with different outcomes; this model serves as the basis for solving one of the most difficult problems of prognosticating - forecasting the development of certain areas of working out a large project with the assessment of the probability of each area and the time of its implementation.

The mathematical methods of modeling the project implementation processes (the classical network models, general and stochastic network models) that are used do not always correspond to the complex realities of the simulated process. This applies to each method separately and some of the interrelations with each other.

With all the advantages of generalized network models in terms of a more flexible and adequate description of the technological and organizational links of the project work, however, they are based on deterministic methods. This leads to systematic errors in the estimates of network parameters, and in the assessment of the project execution time, in particular. Probability and stochastic network models with "classical" ways of describing network topology are too complex, they are not flexible, do not suit the reality and, consequently, are not effective in control.

Modeling processes for projects is an effective methodological core of Project Management. The efficiency of decision-making and, ultimately, the success of the project depend on how the models suit the real processes and requirements addressed in the Project Management activities. Creating single (unique) objects requiring multiple operations in complex technological interdependencies between operations necessitates the development of qualitatively different models and methods to manage them. This direction in research and their practical applications formed the basis for the modern management concepts, the core of “Project Management” (Project management) [1, 2].

In essence, the project is a set of logically related actions to achieve one or more objectives. Therefore, the model, which describes this set of actions, must reflect both the actions with their characteristics and complex logical relationships between the activities. The basis for the logical relationships is made up of technological dependences between some actions or the availability of resources.

The method based on network models (directed graphs [7]) is most convenient in displaying logical relationships. We will present the model development process as a cyclic alternative network model (CANM) [11].

This class of models is a combination of generalized network models with their extensive range of features of equivalent transformations of models and a description of the logical and temporal relationships between the structural elements of the project, with probability and alternative models, which to a great extent are concerned about the risks factor and uncertainties in the implementation of the project. These models are the most flexible and appropriate tools of modeling complexes of discrete operations and describing the process of managing the implementation of a complex and comprehensive project.
The figure presents CANM with the conditions under which it will transform into a traditional, generalized, probability and alternative models, thus being their logical synthesis.

**Description of the Cyclic Alternative Network Models**

**TRADITIONAL MODEL**

$\Psi_{ij}$ - positive constants, $p_{ij} = \{0,1\}$

Connections only “end-beginning”

**GENERALIZED MODEL**

$\Psi_{ij}$ - constants from $-\infty$ to $+\infty$, $p_{ij} = \{0,1\}$

Generalized connections (including the reciprocal ones) between any points of events

**CYCLIC ALTERNATIVE NETWORK MODELS (CANM)**

CANM - the end, oriented, cyclic graph $G(\Omega,A)$, consisting of multiple events $\Omega$ and arcs $(i,j)$ $(i,j,\Omega)$, determined by a matrix of blank $A = \{p_{ij}\}$, $0 \leq p_{ij} \leq 1$, $p_{ij}$ sets the determined arc $(ij)$, but $0 \leq p_{ij} \leq 1$ determines an alternative event $i$ which with probability $\Pi_{ij}$ is connected by the arc with event $j$.

$T_i - T_j \geq \Psi_{ij}$  \hspace{1cm} (1)

where $T_i$ - the time of event $i$ completion, $\Psi_{ij}$ in a general case is any amount divided by a certain law from $-\infty$ to $0$ or from $0$ to $+\infty$. This provides the generalized, probability, and alternative technological connections (including the reciprocal ones) between any points of events. Besides, there can occur absolute limitations during the event $i$ realization:

$l_i \leq T_i \leq L_i$  \hspace{1cm} (2)

**PROBABILITY MODEL**

$\Psi_{ij}$ - positively divided any amount (generally beta-division), $p_{ii} = \{0,1\}$

Connections only “end-beginning”

**ALTERNATIVE MODEL**

$\Psi_{ij}$ - any amount divided by a certain law in the interval from $0$ to $+\infty$, $0 \leq p_{ij} \leq 1$

Connections only “end-beginning”
Many interrelated goals that reflect the structure of the project and its participants may be built for each project [1]. To be able to determine the extent to which the objectives of the project have been completed, we are going to choose the appropriate criteria (optimal schedule). Based on these criteria, we will evaluate the alternative solutions to achieve the goals of the project.

The criteria of scheduled plan optimization can be divided into two groups depending on whether the specified completion deadlines of the project and its individual stages have been set.

The first group includes the criteria that reflect the compatibility of performance time with the set deadlines:

- minimizing the total, or the maximum or mean missing the set deadlines
- minimizing the costs associated with the failure of the stages of work-in-time (penalties for late delivery made at intermediate stages of production, loss on machine downtime or equipment, the damage due to deterioration of a company’s reputation, excessive transport costs for express delivery of delayed products, etc.)
- optimizing the quality indicator of resources’ use (irregularity of their consumption, costs associated with a deficit or surplus of core resources)
- minimizing the number of events missing the deadline, etc.

The criteria of the second group are made up when the deadlines have not been set and are generally based on the total duration of the project realization:

- minimizing the duration of the project’s process
- minimizing downtime of the performers
- minimizing the average timing of certain work stages controlled (and paid) by the customer
- maximizing the indicator of the use of equipment (and other resources)
- minimizing the costs of work in progress
- minimizing the costs of equipment changeovers, etc.

Along with the common criteria, we also have to use the average length of the project’s completion (i.e., the time between the development of the project’s concept and its completion), as well as the average number of projects in production, the average time of waiting in lines, etc. There also occur specific criteria; for example, in the conditions of stopping the project, the duration of the liquidation period is minimized; at this time the final processing of all projects and labor (and equipment) are available for other applications.

Sometimes the optimality criterion is the sum of costs, including the costs of pre-project preparation, the costs of maintaining inventory, losses due to shortages, costs of general and overtime pay; the costs considered are the ones discounted at the beginning (or end) of the planning period.
The optimality criteria described earlier constitute the general problem of scheduling in industry. The project management is characterized by the project’s completion deadlines and the costs of implementation.

In [11] the formulation and algorithms for solving the following problems are considered:

- minimizing the time of the project’s completion under the limitations of resources
- minimizing the quality of resource consumption for a given time of the project’s completion
- optimal allocating limited resources to CANM with variable intensity of the activities
- making up a plan of minimal costs

The proposed algorithms which consider the risk (the dispersion of the timing of events’ completion) by using CANM make it possible to analyze and build consistent models of project completion process, which, in turn, contribute to making up optimal schedules for their implementation. Complex projects are characterized by high costs and the long term of implementation. During project realization, significant funds excluded from active participation in the business are kept in reserve for a long time. Due to the temporary diversion of funds in the project, the amount of "loss" must be taken into account.

Therefore, the present estimated cost and the present cost of the project must be taken as key indicators instead of the estimated cost and the cost price. This means that in order to estimate the time factor, with the use of the discount quotient, all the costs must be discounted to a single point (the end of the project).

At present, while evaluating project investment planning, the DCF (discounted cash flow) methods are widely used. The following is a description of setting the problem of optimal scheduling of the project by using DCF methods.

The essence of the DCF methods is discounting cash flows of different times (costs and benefits) to a point in time, which makes it possible to compare the different time costs and benefits. Based on different ways of comparing the mentioned costs and benefits, a system of indicators to measure the effectiveness of the projects has been created:

- net present value (NPV)
- profit index (PI)
- Internal Rate of Return (IRR), etc.

The most popular is the net present value of profit calculated as the difference between the discounted results and costs, i.e.

\[ \text{NPV} = R^d - C^d \]

where:

- \( R^d \) - net inflow of operating activities discounted to a single point in time,
- \( C^d \) - the investment costs discounted to the same time.

If \( \text{NPV} \geq 0 \), the project is considered effective and it can be recommended for implementation.
When comparing different options of the project realization, we should prefer the version having the greatest value NPV, i.e. the objective function of the scheduling problem, is presented as follows:

$$f = R^d - C^d \rightarrow \text{max.} \quad (3)$$

In general, when scheduling the implementation of investment projects it is possible to affect the value of both $R^d$ and $C^d$. However, in most situations, the main possibilities of maximizing $R^d$ become exhausted while forming the project domain (for example, through the provision of start-up queues or complexes). The scheduling of the project mainly covers the investment phase of the project. It can, therefore, be quite reasonable to assume that the scheduling problem holds $\text{max} (R^d - C^d) \Leftrightarrow \text{min} C^d$.

The idea of using the presented costs for evaluating the capital building projects (presented point - the date of commissioning the project) was proposed by Academician L.V. Kantorovich in 1965 [9]. The methods of optimizing the scheduling problems in the classical network formulation of the reduced costs were first developed by L.I. Averbakh in 1968. [10].

The above criteria and methods to optimize schedules based on DCF indicators were proposed more than three decades ago. They have not found practical application in the socialist model of the economy, as the economic conditions of the project of both a customer and contractor(s) not only did not encourage them to optimize activities by these criteria, but, as a rule, forced them to conduct economic policy in the opposite direction.

In the civilized market economy, which Russia has assumed, NPV is a key leader in the economic evaluation of investment policy of economic entities; it is of strategic importance. In this paper, we use DCF criteria to set scheduling problems based on CANM.

Discounted by the end of the project, its final cost is determined by the formula:

$$C^d = \sum_{t=1}^{T} C_t (1 + d)^{T-t}, \quad (4)$$

where $T$ - the duration of the project, $C_t$ - the value of the used funds in the period $t$, $(t = 1, 2, ..., T)$, $d$ - the discount factor.

Let $A(t) = \sum_{1}^{t} C_t$, $t = 1,2, ..., T$. Piece-wise linear function $A(t)$ shows an increase of capital expenditures over time during the project from $0$ at $t = 0$ and $C$ at $t = T$, where $C$ - the estimated cost of the project.

The size of diverted funds is determined by

$$S = \int_{0}^{T} A(t) \, dt \quad (5)$$
Setting the ratio of diverting the funds to the estimated value as $\tau$, we have

$$\tau = \frac{S}{C}$$

(6)

The value $\tau$ is the average period of diverting funds to the unfinished project, in other words, $\tau$ indicates for how long each ruble of capital investments during the project is diverted.

Then (4) can be represented as follows:

$$C^d = C(1 + d)^\tau.$$

(7)

Since $C$ and $d$ are constants, it is obvious that the optimal plan of the project in terms of "present value" is the one in which $\tau$ reaches a minimum value (subject to the technological sequence of work and rational limits of intensity of their implementation).

[10] refers to the conditions under which $\tau$ reaches a minimum value.

As a result of the analysis of these conditions, the following theorem is proved:

*The minimum value of the diverted funds is achieved when all of the activities are performed with a minimum length (maximum intensity) at a later time. Hence, the schedule, providing a minimum amount of diverted funds corresponds to the minimum duration of the project.*

These results are the basis of the proposed algorithm of forming a schedule, optimal by the criterion of the discounted costs.

Thus, the scheduling problem in project management can be formulated as follows.

**Given:** a list of planned projects and characteristics of activities involved (volume, complexity, cost, workers, etc.). The total number of resources available to any segment of the planning period, is known, along with the area of possible use of each resource, performance, and the intensity limits of their use in each work (or part of it).

It is required to make up a schedule (timetable of activities), to meet all the conditions of the problem. Making up such a plan equals to the definition of the unknowns - the times of activities’ completion $T_i$, corresponding to the dates of the start and finish of activities or their parts. The set of numbers $T = \{T_i\}, \ i = 1, 2, ..., n,$ is a valid plan $T$, if the above conditions and constrains of the problem are met. In addition, the constraints can be meeting the requirements of specified parameters, which characterize the reliability of adopted organizational, technological, and economic decisions.

A certain numerical (objective) function $f(T)$ is defined for each optimality criterion, determined on all plans as $T$, and the extreme value of which defines the best plan.
Thus, the general problem of optimal scheduling the project implementation process is to determine the schedule of the activities $T$ meeting all the conditions set forth in the problem and the constraints. In addition, in this plan the objective function $f(T)$ reaches its extreme value.

The optimality criterion reflects the essential characteristic of the plan and can be made up based on any points mentioned in [11] or those of the above, according to different conditions and objectives of a specific project. Some other requirements may be part of the problem as constraints.

The attempts to apply the methods of stochastic planning based on the classic network models for solving the above problem have not yielded the expected results in practice. This occurred primarily because each planner seeks to identify the uncertainties in the early stages of the project and to find suitable alternatives. Then the decision is made in favor of the alternative with the highest subjective probability, and the further process is considered as a determined model of the project.

The synthesis of stochastic and generalized network models proposed in [11] and used in this paper can relieve some of the above issues by making the generalized models more flexible and providing them with information of different (for each phase and planning level) degree of aggregation. The use of simulation modeling and some of the ideas of the theory of expert systems give us hope that the life span of the proposed approach and the possibility of its practical use are long enough.

Therefore, by using the proposed model and the appropriate methods of obtaining optimal (in some sense) making up schedules we have to be able to answer the following questions of project managers:

- What is the most likely timing of work that provides an optimum value of the objective function?
- What is the probability that the project will be carried out at a longer time?
- What is the most likely time of completion?
- What is the probability that the calculated date of an event’s fulfillment is not affected?
- What date of a specific event, that concerns the project manager, will not be surpassed with a given probability $p$ (ex. 0.95 or 0.99)?

**Conclusion**

The research carried out in this paper provides a theoretical and methodological basis for solving problems of planning and project management and allows for the following:

- The use of the universal model (CANM) provides for the ability to consider the alternative character of both the technology of activities and the means of assigning resources at optimal rate of use. Thus, the methods of resource and
Timing analysis discussed can be effectively used in managing complex projects, considering the risk and uncertainty in the environment in which they are running.

- These models and associated algorithms can be used if the object of management is a complex of projects and, if required, each project individually or as a set of projects is implemented in the shortest possible time.

- The making up optimized implementation schedules, as well as an optimized master plan for the complex projects that involve the proposed models and algorithms, provides for an opportunity to determine the appropriate resource requirements (including financial), schedules for performers, and the use of machines and equipment.

- Periodic updating of the initial data allows the specialists to specify these requirements and schedules, thereby reducing uncertainty and creating the necessary conditions to improve the implementation of projects in the space of the "time-to-value resources."

- The fact of using the universal modeling tools and algorithms to solve scheduling project problems can expand the scope of applying network modeling techniques to managing such projects, the complexity of which is caused by a great variety of technological interdependencies of various activities, their probabilistic nature of the relationship and the parameters carried out in the face of uncertainty.

References


About the Authors

Lev I. Averbakh, PhD

New York, USA

Lev Averbakh was born in Chelyabinsk, Russia in 1937. In 1959 he graduated with Master Degree in Industrial Construction from Chelyabinsk Polytechnical Institute (currently South-Ural State University). Later, he worked as CEO of a large construction company and as senior project manager overseeing reconstruction of the largest in Russia Tractor Factory located in the city of Chelyabinsk. In 1968 Lev completed his term as a research fellow in Novosibirsk State University in the area of Economical Cybernetic, and received a PhD in Economics. In the early seventies Lev created and managed a company that was working on development of automatic control systems and management information systems, also he functioned as chief engineer of these systems. Since 1987 Lev had been working as a professor and had been teaching following subjects: Fundamentals of Management; Economic Analysis of Investment Projects; Mathematical Methods of decision-making; Financial mathematics; Economic Theory; and Securities Market and Trading. Lev has published about 50 research papers and has made numerous conference presentations. In 2001 he moved to the USA and continued his scientific research in cooperation with Russian colleges.

Yan D. Gelrud, PhD

South Ural State University

Chelyabinsk, Russia

Mr. Yan Gelrud was born in 1947 in Birobidjan (Khabarovsk Territory). In 1965 he finished a school of mathematics and physics at Novosibirsk. In 1970 he graduated from the mathematical faculty of university at Novosibirsk on "Mathematics" speciality. From 1970 to 1991 Yakov was working in the Research Institute of automated control systems as a head of mathematical division. He
took part in creation and adoption of more than 100 automated control systems in different branches of industry.

From 1991 to 1997 Mr. Gelrud was doing business, being director general of "URAL-ASCO-SERVICE". Since the 1st of September 1997 till now he works as a professor of the "Enterprise and management" department in South Ural State University. He teaches a multitude of disciplines, such as "Mathematics", "Theory of probability and mathematical statistics", "Econometrics", "Economic and mathematical methods", "Mathematical methods of decision-making", "Bases of decision-making methodology", "Economical evaluation of investments", "Mathematical methods and models of project management", "Studies of managerial systems."

Yan Gelrud has more than 100 publications and speeches on seminars and conferences of different level. His monograph "Project management in conditions of risk and uncertainty" was published recently. He can be contacted at gelrud@mail.ru

Vladimir Voropajev, PhD

Author, Professor, International PM Expert
Founder, Former President, Chair – SOVNET
Former Vice President – IPMA
Full Member, Russian Academy of Natural Sciences

Moscow, Russia

Professor Vladimir Voropajev, PhD. is Founder and former President and Chairman of the Board of the Russian Association of Project Management, SOVNET. Dr. Voropajev is professor of Project Management at the State University of Management, Moscow, Russia. He is also Head of the Program and Project Management Faculty for the Russian State Academy’s Program for Professional Retraining and Professional Skill Development for Executives and Specialists in Investment Fields. He is a full member of the Russian Academy of Natural Sciences on Information Science and Cybernetics, and of the International Academy of Investments and Economy in Construction. From 1991 to 2001, he was Vice-president and a member of the Executive Board of the International Project Management Association (IPMA), the global federation of national PM associations based in Zurich, Switzerland. He is the First Assessor for several IPMA certification bodies. In 2005 he was awarded IPMA Honorary Fellowship Award. He is also an honorary Fellow of the Indian Project Management Association and a past member of the Global Project Management Forum Steering Committee. During his 40 years of engineering, scientific, teaching and consulting activities, he has published over 250
scientific research works including 7 monographs and 5 textbooks about the organization and planning of construction, information systems, and project management. Vladimir serves on the editorial boards of several international project management journals, is a frequent participant in PM conferences worldwide, and provides ongoing counsel and support to PM professional leaders in Azerbaijan, Kazakhstan, Ukraine, Yugoslavia and several other countries. Professor Voropajev can be reached at voropaev@sovnet.ru